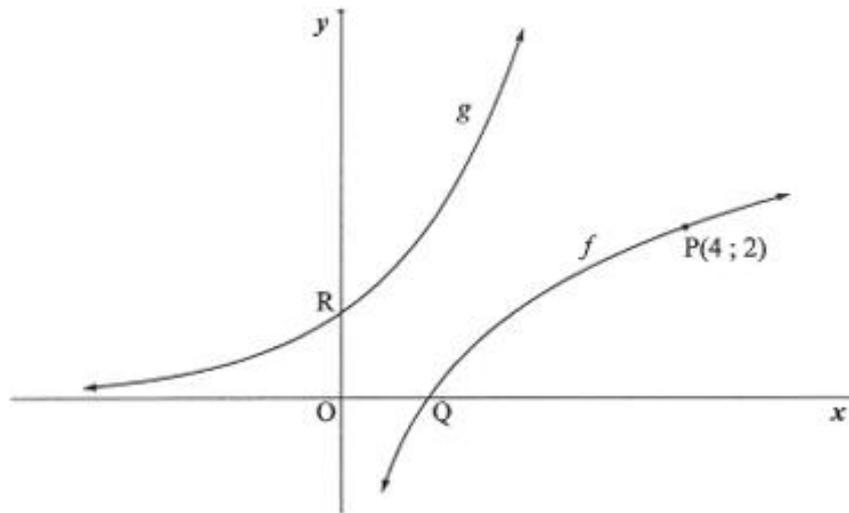**FUNCTIONS AND INVERSES**

MAY/JUNE 2024

QUESTION 5

In the diagram, the graphs of $f(x) = \log_a x$ and g are drawn. Graph g is the reflection of f in the line $y = x$. Graph f passes through the point $P(4 ; 2)$. Q is the x-intercept of f and R is the y-intercept of g.
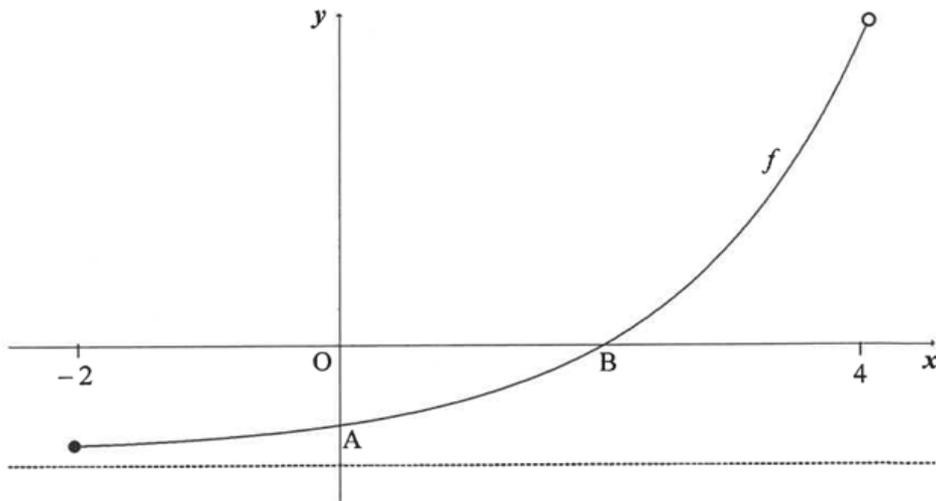

5.1   Write down the coordinates of P', the image of P on g.   (2)

5.2   Show that $a = 2$.   (2)

5.3   Write down the equation of g in the form $y = \ldots$ (1)

5.4   T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of $\triangle RTP'$.   (4)

[9]

QUESTION 4

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2 \, ; 4)$.
A and B are respectively the y- and x-intercepts of f.

4.1	Write down the equation of the asymptote of f.	(1)
4.2	Determine the coordinates of B.	(2)
4.3	Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$	(3)
4.4	Calculate the vertical distance between k and f at $x = 1$	(3)
4.5	Write down the equation of g if it is given that $g(x) = f(x) + 4$	(1)
4.6	Write down the domain of g^{-1}.	(2)
4.7	Write down the equation of g^{-1} in the form $y = \ldots$	(2)

[14]

QUESTION 4

4.1   Given the function $p(x) = \left(\dfrac{1}{3}\right)^x$.

        4.1.1   Is p an increasing or decreasing function?   (1)

        4.1.2   Determine p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

        4.1.3   Write down the domain of p^{-1}.   (1)

        4.1.4   Write down the equation of the asymptote of $p(x) - 5$.   (1)

4.2   Given: $f(x) = \dfrac{4}{x-1} + 2$

        4.2.1   Write down the equations of the asymptotes of f.   (2)

        4.2.2   Calculate the x-intercept of f.   (2)

        4.2.3   Sketch the graph of f, label all asymptotes and indicate the intercepts with the axes.   (4)

        4.2.4   Use your graph to determine the values of x for which $\dfrac{4}{x-1} \geq -2$.   (2)

        4.2.5   Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient.   (3)

                                                                [18]

4.2   The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.
- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.
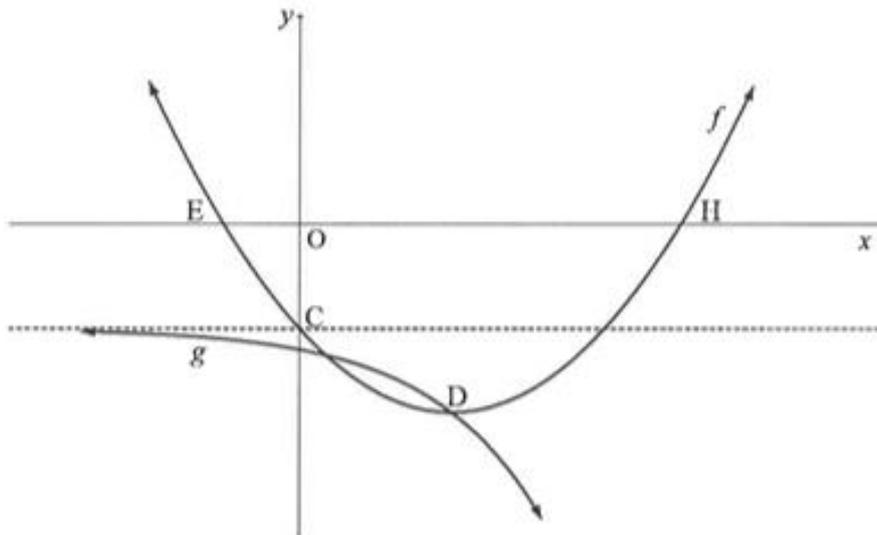
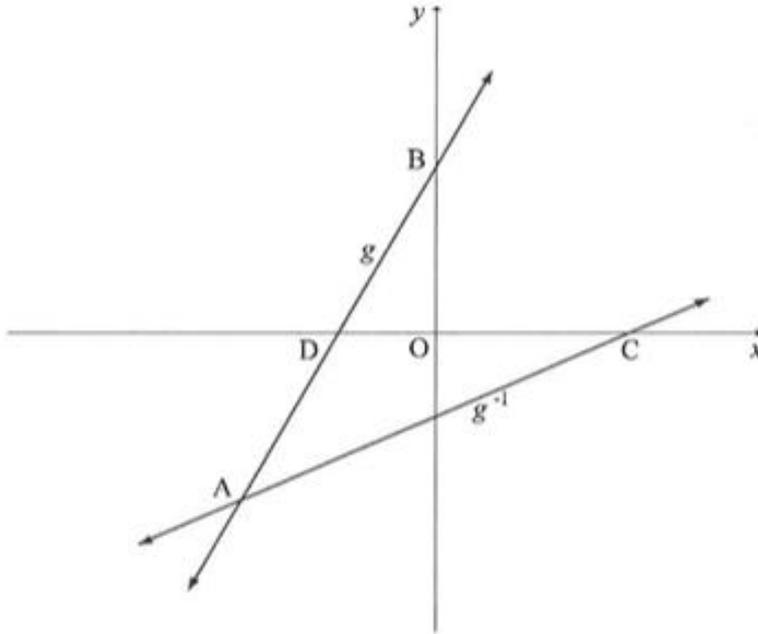4.2.1   Write down the y-coordinate of C.   (1)

4.2.2   Determine the coordinates of D.   (2)

4.2.3   Determine the values of a and q.   (3)

4.2.4   Write down the range of g.   (1)

4.2.5   Determine the values of k for which the value of $f(x) - k$ will always be
          positive.   (2)

                                                                                               [20]

QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.
- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.

| 5.1	Write down the y-coordinate of B.	(1)
5.2	Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$.	(2)
5.3	Determine the coordinates of A.	(3)
5.4	Calculate the length of AB.	(2)
5.5	Calculate the area of $\triangle ABC$.	(5)
		[13]

QUESTION 6

6.1   Given: $g(x) = 3^x$

      6.1.1   Write down the equation of g^{-1} in the form $y = \ldots$ (2)

      6.1.2   Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2   Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.
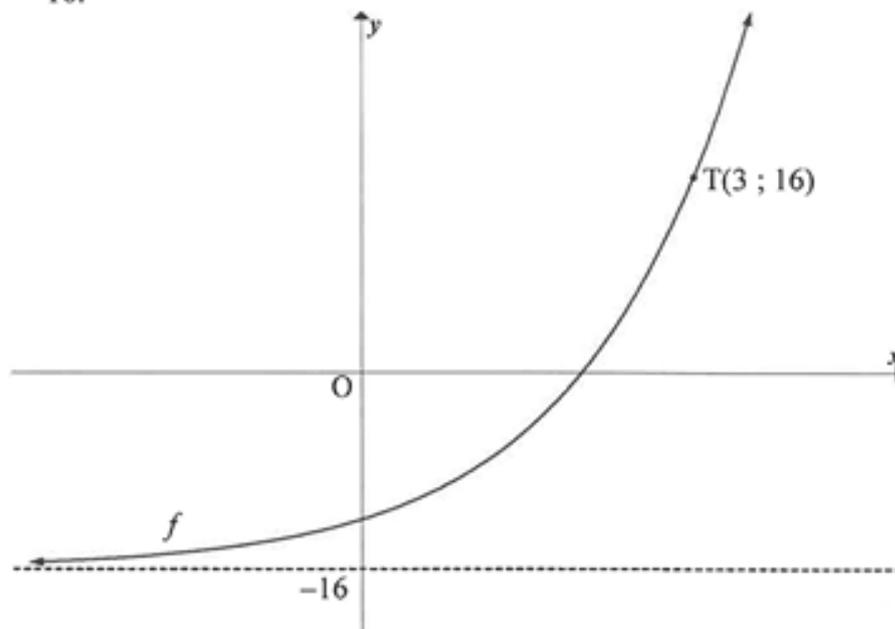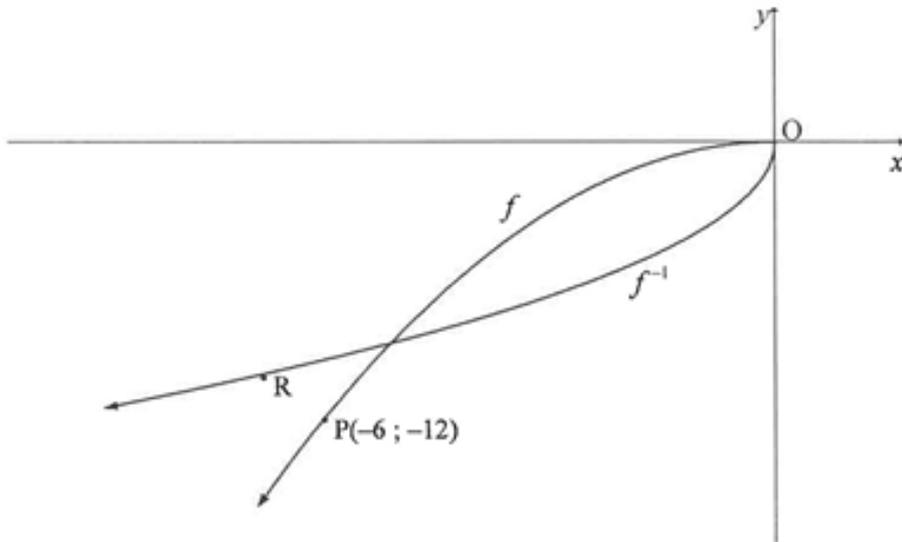

Determine the values of p and q. (4)

[8]

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.
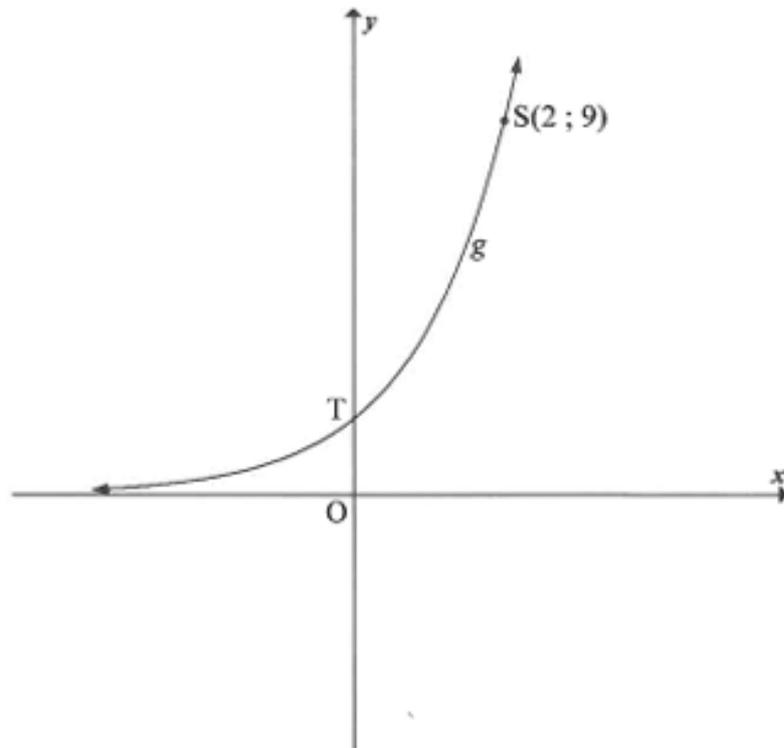

4.1	Is f^{-1} a function? Motivate your answer.	(2)
4.2	If R is the reflection of P in the line $y = x$, write down the coordinates of R.	(1)
4.3	Calculate the value of a.	(2)
4.4	Write down the equation of f^{-1} in the form $y = ...$	(3)
		[8]

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.

5.1   Write down the coordinates of T.   (2)

5.2   Calculate the value of a.   (2)

5.3   The graph h is obtained by reflecting g in the y-axis. Write down the equation of h.   (2)

5.4   Write down the values of x for which $0 < \log_3 x < 1$.   (2)
                                                                                                                            [8]

QUESTION 6

The function f, defined by $f(x) = \dfrac{a}{x+p} + q$, has the following properties:

- The range of f is $y \in R,\ y \neq 1$.
- The graph f passes through the origin.
- $P\left(\sqrt{2} + 2; \sqrt{2} + 1\right)$ lies on the graph f.

6.1   Write down the value of q.   (1)

6.2   Calculate the values of a and p.   (5)

6.3   Sketch a neat graph of this function. Your graph must include the asymptotes, if any.   (4)

[10]